Hunters Hill High School

Mathematics

Trial Examination, 2018

Hunters Hill
High School

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using only black pen
- Board-approved calculators may be used
- A reference sheet is provided
- The marks for each question are shown on the paper
- Show all necessary working in questions 11-16

Total Marks: 100

Section I Pages 3-5
10 marks

- Attempt Questions 1-10
- Allow about 20 minutes for this section

Section II Pages 6-12
90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 40 minutes for this section

Section I

10 marks Attempt Questions 1–10
Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. What is the value of $\frac{\pi^3}{6}$, correct to 3 significant figures?

(A) 5.168
(B) 5.167
(C) 5.16
(D) 5.17

2. Which line is perpendicular to the line $3x + 4y + 7 = 0$?

(A) $3x - 4y + 7 = 0$
(B) $8x - 6y - 7 = 0$
(C) $4x + 3y - 7 = 0$
(D) $4x - 7y + 7 = 0$

3. William has five pairs of socks, all of different designs.
If he selects two socks at random, what is the probability that they form a matching pair?

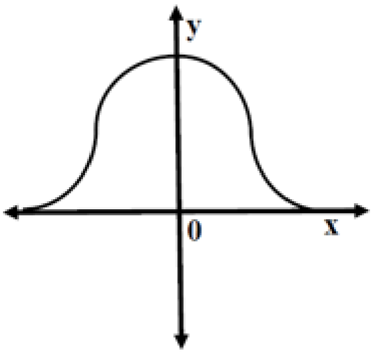
(A) $\frac{1}{2}$
(B) $\frac{1}{9}$
(C) $\frac{5}{9}$
(D) $\frac{1}{10}$

4. The domain of the function $f(x) = \sqrt{x^2 - 1}$ is ?

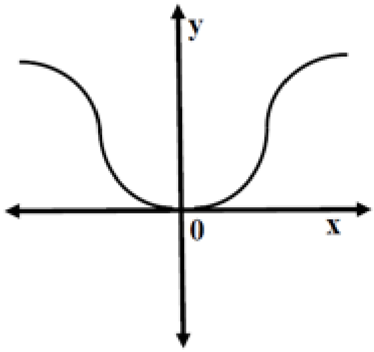
(A) $x \geq 1, x \leq -1$
(B) $-1 \leq x \leq 1$
(C) $x \geq 1$
(D) $x \leq -1$

5. Which graph best represents $y = 1 - \sin\left(x - \frac{\pi}{2}\right)$ in the domain $-\pi \leq x \leq \pi$?

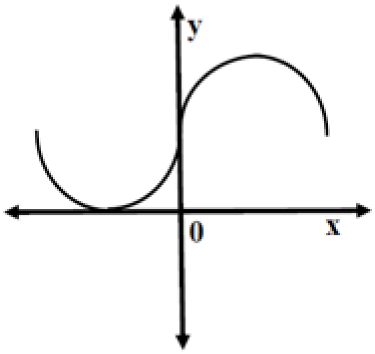
(A)



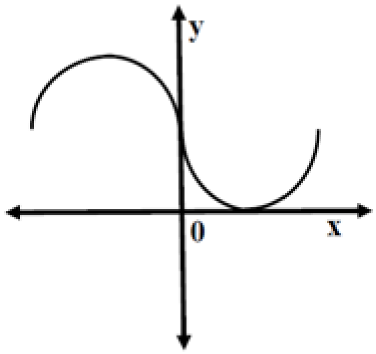
(B)



(C)



(D)



6. The function $f(x)$ is given by $f(x) = x^3 - 27x$. The coordinates of the minimum stationary points are ?

(A) $(3, -54)$

(B) $(-3, 54)$

(C) $(-3, -54)$

(D) $(3, 54)$

7. What is the solution to $3^x = 2$?

(A) $x = \frac{\log_e 2}{3}$

(B) $x = \frac{3}{\log_e 2}$

(C) $x = \frac{\log_e 2}{\log_e 3}$

(D) $x = \log_e\left(\frac{2}{3}\right)$

8. A point $P(x, y)$ moves such that it is 4 units from the point $(-1,2)$. The equation of the locus of $P(x, y)$ is ?

(A) $(x+1)^2+(y-2)^2=4$

(B) $(x+1)^2+(y-2)^2=16$

(C) $(x-1)^2+(y+2)^2=16$

(D) $(x-1)^2+(y+2)^2=4$

9. For an arithmetic sequence, the sum to n terms is given as $S_n = 3n - 2n^2$.
The $10th$ term is ?

(A) -30

(B) -25

(C) -40

(D) -35

10. A population of sea monkeys is observed to fluctuate according to the equation $\frac{dP}{dt} = 40\sin(0.1t)$, where P is the sea monkey population and t is the time in days.
During which day does the population first start to decrease?

(A) Day 15

(B) Day 16

(C) Day 31

(D) Day 32

End of Section I

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 40 minutes for this section
Begin each question on a NEW SHEET of paper.

In questions 11 – 16 your responses should include relevant mathematical reasoning and/or calculations.

Question 11 – Use a NEW SHEET of paper. (15 marks)

a. Rationalise the denominator $\dfrac{1-\sqrt{3}}{5-\sqrt{3}}$ **2**

b. Factorise $2x^2 - 17x + 30$ **2**

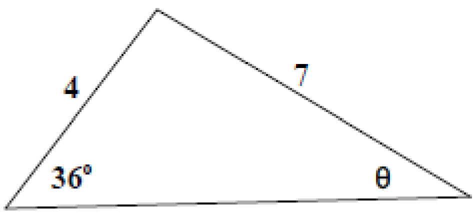
c. Differentiate $y = 4x^3 - \sqrt{x}$ **2**

d. Find $\displaystyle\int \left(x^3 - \frac{2}{x}\right) dx$ **2**

e. Find $\displaystyle\int_0^{\pi} \sin 2t \; dt$ **2**

f. Solve $|2x+1| > 6$ **3**

g. Find the value of θ, correct to the nearest degrees. **2**

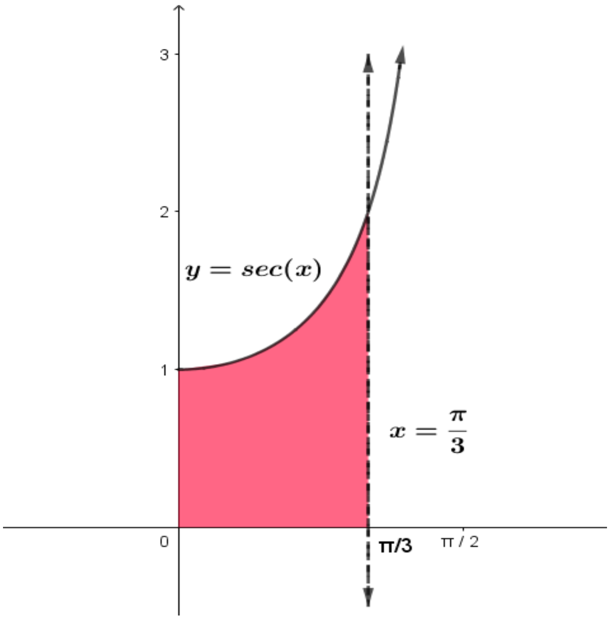


End of Question 11

Question 12 – Use a NEW SHEET of paper. (15 marks)

a. Find the equation of the tangent to the curve $y = x^2 + 4x - 7$ at the point $(1, -2)$. **2**

b. What is the exact volume of the solid of revolution formed by rotating the curve $y = \sec x$ about the x-axis, for $0 \leq x \leq \frac{\pi}{3}$? **3**



c. (i) Evaluate $\int_0^1 \sin \pi x \, dx$, leaving your answer in exact form. **2**

(ii) Copy and complete, using exact values, the table below for the function $y = \sin \pi x$. **2**

x	0	0.25	0.5	0.75	1
y					

Using Simpson's Rule with five function values, give an estimate for the area under the curve in part (i).

(iii) Using part (i) and (ii) give an estimate for π correct to two decimal places. **2**

Question 12 continues on next page

d. A standard pack of 52 cards consists of four suits (Diamonds, Hearts, Clubs and Spades) with 13 cards in each suit.

(i) One card is drawn from the pack and kept on the table. A second card is drawn and placed beside it on the table.
What is the probability that the second card is from a different suit to the first? **2**

(ii) The two cards are replaced and the pack is shuffled. Four cards are chosen from the pack and placed side by side on the table without replacement.
What is the probability that these four cards are all from different suits? **2**

End of Question 12

Question 13 – Use a NEW SHEET of paper. (15 marks)

a. Evaluate $\lim_{x \to -4} \dfrac{x^2 + 4x}{x + 4}$ **2**

b. Given the function, $f(x) = x^4 - 2x^3$

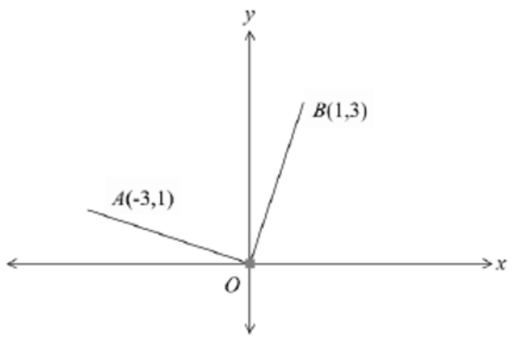
(i) Find the coordinates of the points where the curve crosses the axes. **2**

(ii) Find the coordinates of the stationary points and determine their nature. **2**

(iii) Find the coordinates of the points of inflexion. **2**

(iv) Sketch the graph of $y = f(x)$, clearly indicating the intercepts, stationary points and points of inflexion. **2**

c. Points $A(-3, 1)$ and $B(1, 3)$ are on a number plane.



(i) Find the gradient of line OA. **1**

(ii) Show that OA is perpendicular to OB. **1**

(iii) OACB is a quadrilateral in which BC is parallel to OA. **2**
Show that the equation of BC is $x + 3y - 10 = 0$.

(iv) The point C lies on the line $x = -2$.
What are the coordinates of point C? **1**

End of Question 13

Question 14 – Use a NEW SHEET of paper. (15 marks)

a. Sketch the curve $y = 2\sin 2\left(x + \frac{\pi}{2}\right)$ in the domain $-\pi \leq x \leq \pi$. **3**

b. The velocity of a particle is given by $\frac{dx}{dt} = \frac{t}{t^2 + 1}$ m/s.

If the particle is initially at the origin, find the displacement after 4 seconds.

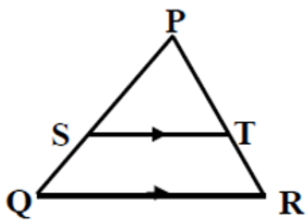
Give your answer correct to 3 significant figures. **3**

c. **(i)** Differentiate $y = \sqrt{4 - x^2}$ with respect to x. **2**

(ii) Hence, or otherwise, find $\int \frac{3x}{\sqrt{4 - x^2}}\,dx$. **2**

d. The diagram below shows the ΔPQR.

ST is parallel to QR. $PT = 6$, $TR = 4$ and $PQ = 12$.



(i) Prove that ΔPST is similar to ΔPQR. **3**

(ii) Find the length of SQ. **2**

End of Question 14

Question 15 – Use a NEW SHEET of paper. (15 marks)

a. Solve for x the equation $4^x - 10 \times 2^x + 16 = 0$. **2**

b. The roots of the quadratic equation $(m+2)x^2 + (m-2)x - 2 = 0$ are equal in magnitude but are opposite in sign.

(i) Find the value of m. **2**

(ii) Find the value of the roots. **2**

c. Find all values of θ in the domain $0 \leq \theta \leq 360°$ for which $\sec\theta - 2\cos\theta = 0$. **3**

d. Jill wishes to have \$900000 in her fund when she retires in 10 years time. At present she has \$400000 in her fund. For the next 10 years she decides to put \$M into her fund at the beginning of each month. The contributions attract an interest rate of 6% per annum, compounding monthly.

At the end of n months after starting the contributions, the amount in the fund is $\$A_n$.

(i) Show that $A_1 = 400000 \times 1.005 + 1.005M$. **1**

(ii) Show that $A_2 = 400000 \times (1.005)^2 + (1.005 + 1.005^2)M$. **2**

(iii) Find the value of M so that Jill will have \$900000 in the fund after 10 years. **3**

End of Question 15

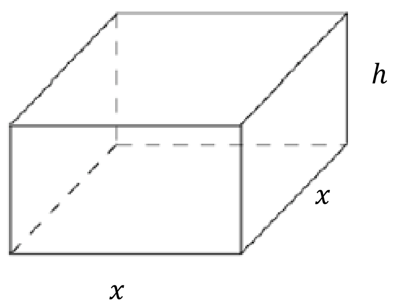
Question 16 – Use a NEW SHEET of paper. (15 marks)

a. The number of bacteria N in a colony, where t is in minutes, is given by $N = 1000e^{0.005t}$. Find:

(i) The number of bacteria when $t = 20$. **1**

(ii) The rate at which the colony is increasing when $t = 20$. **2**

b. The material for the square base of a rectangular box with an open top costs 27 cents per square cm and for the other faces costs 13.5 cents per square centimetre.



(i) Show that the total cost of materials, C, for the box can be written as **1**

$$C = 27x^2 + 54xh.$$

(ii) If the cost of making each box is \$65.61, find an expression for h in terms of x. **1**

(iii) Show that the formula for the volume of the box can be expressed as **2**

$$V = \frac{1}{2}(243x - x^3).$$

(iv) Find the maximum volume of a box that can be produced for \$65.61. **3**

c. The displacement x cm of a particle from the origin after t seconds is given by $x = 2t - \ln t$.

(i) Determine the position and acceleration of the particle when it comes to rest. **3**

(ii) What is the limiting velocity and limiting acceleration that the particle approaches as t increases. **2**

End of paper

HHHS 2016 MAT TRIAL

Page 1

Sunday, 29 July 2018 [illegible] PM

Q1 $\frac{\pi^3}{6} = 5.16771278\ldots = 5.17$ (D)

Q2 $3x + 4y + 7 = 0$

$4y = -3x - 7$

$y = \frac{-3}{4}x - \frac{7}{4}$

$m = \frac{-3}{4}$ perpendicular $m_1 m_2 = -1$

$m_2 = -1 \div \frac{-3}{4}$

$m_2 = \frac{4}{3}$

$8x - 6y - 7 = 0$

$-6y = -8x + 7$

$y = \frac{4}{3}x - \frac{7}{6}$ (B)

Q3 $P(MP) = 1 \times \frac{1}{9} = \frac{1}{9}$ (B)

Q4 $x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq 1$ or $x \leq -1$ (A)

Q5 (A) phase shift $\frac{\pi}{2}$ to the left

Q6 $f'(x) = 0 = 3x^2 - 27$

$x^2 - 9 = 0$

$x = \pm 3$ (A)

$f''(x) = 6x > 0$ for min

$\therefore x = 3,\ f(x) = 3^3 - 27 \times 3 = -54$

page 2

Q7 $3^x = 2$

$\log_3 2 = x$ (C)

$x = \frac{\log_e 2}{\log_e 3}$

Q8 $(x - (-1))^2 + (y - 2)^2 = 4^2$ (B)

$(x+1)^2 + (y-2)^2 = 16$

Q9 $S_n = 3n - 2n^2$

$n = 10$ (D)

$T_n = S_n - S_{n-1}$

$= (3 \times 10 - 2 \times 10^2) - (3 \times 9 - 2 \times 9^2)$

$= -35$

Q10 $\frac{dP}{dt} = 0$

$40 \sin(0.1t) = 0$

$\sin(0.1t) = 0$ (D)

$0.1t = \pi$

$t = 31.415$ days

$\therefore$ Day 32

page 3

Q11 a) $\frac{1-\sqrt{3}}{5-\sqrt{3}} = \frac{1-\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$
$= \frac{5+\sqrt{3}-5\sqrt{3}-(\sqrt{3})^2}{5^2-(\sqrt{3})^2}$ } 1 mark
$= \frac{2-4\sqrt{3}}{22} = \frac{1-2\sqrt{3}}{11}$ 1 mark

b) $2x^2-17x+30$
$=(2x-5)(x-6)$

$2x \quad -5$
$x \quad -6$

1 mark if show working towards ans.
1 mark correct ans

c) $y = 4x^3 - \sqrt{x}$
$= 4x^3 - x^{\frac{1}{2}}$ 1 mark
$y' = 12x^2 - \frac{1}{2}x^{-\frac{1}{2}}$
$y' = 12x^2 - \frac{1}{2}\frac{1}{\sqrt{x}}$ } 1 mark

d) $\int \left(x^3 - \frac{2}{x}\right)dx = \frac{1}{4}x^4 - 2\ln x + C$

1 mark if no C
1 mark correct ans

e) $\int_0^{\pi} \sin 2t \, dt = \left[\frac{1}{2}\cos(2t)\right]_0^{\pi}$ 1 mark
$= \frac{1}{2}(\cos 2\pi - \cos 2\times 0)$
$= \frac{1}{2}(1-1)$ 1 mark correct substitution
$= 0$

page 4

Q1 f. $|2x+1| > 6$

$2x+1 > 6$ or $-(2x+1) > 6$
$2x > 5$ $\quad 2x+1 < -6$
$x > \frac{5}{2}$ $\quad 2x < -7$
$x > 2\frac{1}{2}$ $\quad x < \frac{-7}{2} \quad x < -3\frac{1}{2}$

$\therefore x < -3\frac{1}{2}$ or $x > 2\frac{1}{2}$

1 mark for each correct working
1 mark for correct ans

g. $\frac{\sin\theta}{4} = \frac{\sin 36°}{7}$ 1 mark
$\theta = \sin^{-1}\left(\frac{4\sin 36°}{7}\right)$

$\theta = 19.626$ 1 mark
$\theta \simeq 20°$

page 5

Q12 a) $y = x^2 + 4x - 7$

$y' = 2x + 4$

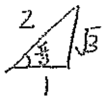
at $(1, -2)$ $y' = 2\times1 + 4 = 6$ — 1 mark for correct gradient

$y - (-2) = 6(x - 1)$

$y + 2 = 6x - 6$

$y = 6x - 8$ or $6x - y - 8 = 0$ — 1 mark

b) $V = \pi \int_0^{\frac{\pi}{3}} y^2\,dx$ — 1 mark correct use of formula and limits



$= \pi \int_0^{\frac{\pi}{3}} \sec^2 x\,dx$

$= \pi \left[\tan x\right]_0^{\frac{\pi}{3}}$ — 1 mark correct integration

$= \pi(\tan\frac{\pi}{3} - \tan 0)$

$= \pi(\sqrt{3} - 0)$

$= \sqrt{3}\pi$ units^3 — 1 mark correct ans

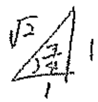
c i) $\int_0^1 \sin \pi x\,dx$

$= \left[-\frac{1}{\pi}\cos \pi x\right]_0^1$ — 1 mark

$= -\frac{1}{\pi}[\cos\pi - \cos 0]$

$= -\frac{1}{\pi}[-1 - 1]$

$= \frac{2}{\pi}$ — 1 mark



ii)

x	0	0.25	0.5	0.75	1
y	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0

1 mark for correct 0.25 and 0.75 values

1 mark for correct 0, 0.5 and 1 values

page 6

ii) Simpson's rule

use the reference sheet

$\int_a^b f(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

apply the rule twice

$a = 0$ $b = 0.5$ $\frac{a+b}{2} = 0.25$

① $f(0) = 0$ $f(0.25) = \frac{1}{\sqrt{2}}$ $f(0.5) = 1$

$\int_0^{0.5} \sin \pi x \simeq \frac{0.5 - 0}{6}\left[0 + 4\times\frac{1}{\sqrt{2}} + 1\right]$

$\simeq \frac{1}{12}\left[\frac{4}{\sqrt{2}} + 1\right]$ units^2

② $f(0.5) = 1$ $f(0.75) = \frac{1}{\sqrt{2}}$ $f(1) = 0$

$\int_{0.5}^1 \sin \pi x \simeq \frac{1 - 0.5}{6}\left[1 + 4\times\frac{1}{\sqrt{2}} + 0\right]$

$\simeq \frac{1}{12}\left[1 + \frac{4}{\sqrt{2}}\right]$ units^2

$\int_0^1 \sin \pi x \simeq \frac{1}{12}\left[\frac{4}{\sqrt{2}} + 1\right] + \frac{1}{12}\left[1 + \frac{4}{\sqrt{2}}\right]$

$\simeq 0.638$ units^2

OR $= \frac{1}{3\sqrt{2}} + \frac{1}{12} + \frac{1}{12} + \frac{1}{3\sqrt{2}}$ — 1 mark for 2 applications

$= \frac{1 + 2\sqrt{2}}{6}$

iii) from (i) $\int_0^1 \sin \pi x\,dx = \frac{2}{\pi} = 0.638$ ← from ii) — 1 mark to make a connection

$\frac{2}{\pi} = 0.638$

$\pi \simeq 3.13$ — 1 mark correct ans

page 7

Q12 d) i) P(2 cards of different suit) $= 1 \times \frac{39}{51}$ 1 mark

$= \frac{13}{17}$ 1 mark

ii) P(4 different suits) $= 1 \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49}$ 1 mark

$= \frac{2197}{20825}$ 1 mark

page 8

Q13 a) $\lim_{x \to -4} \frac{x^2+4x}{x+4}$

$= \lim_{x \to -4} \frac{x(x+4)}{x+4}$ 1 mark for factorise

$= \lim_{x \to -4} x$

$= -4$ 1 mark final ans. no mark if only ans is given

b) (i) $f(x) = x^4 - 2x^3$

x-intercepts: $0 = x^4 - 2x^3$ 1 mark for each point

$x^3(x-2) = 0$

$x = 0$ or $x = 2$

∴ points are $(0,0)$, $(2,0)$

ii) for stationary points $f'(x) = 0$

$f'(x) = 4x^3 - 6x^2 = 0$

$x^2(4x-6) = 0$

$x = 0$ $x = \frac{3}{2}$

$f(0) = 0$ $f(\frac{3}{2}) = (\frac{3}{2})^4 - 2(\frac{3}{2})^3 = \frac{-27}{16}$

$f''(x) = 12x^2 - 12x$

$= 12x(x-1)$

At $x = 0$, $f''(0) = 0$ →

x	-1	0	1
$f''(x)$	+	0	−

∴ $(0,0)$ is a horizontal point of inflecion

At $x = \frac{3}{2}$, $f''(\frac{3}{2}) = 9 > 0$ $f(\frac{3}{2}) = \frac{-27}{16}$

∴ $(\frac{3}{2}, \frac{-27}{16})$ is a minimum turning point

1 mark for each point

page 9

Q13 iii) at the point of inflexion

$f''(x) = 12x(x-1)$

previously, we already found $(0,0)$ is a point of inflexion. 1 mark

$x - 1 = 0$

$x = 1 \quad f(1) = 1^4 - 2\times1 = -1$

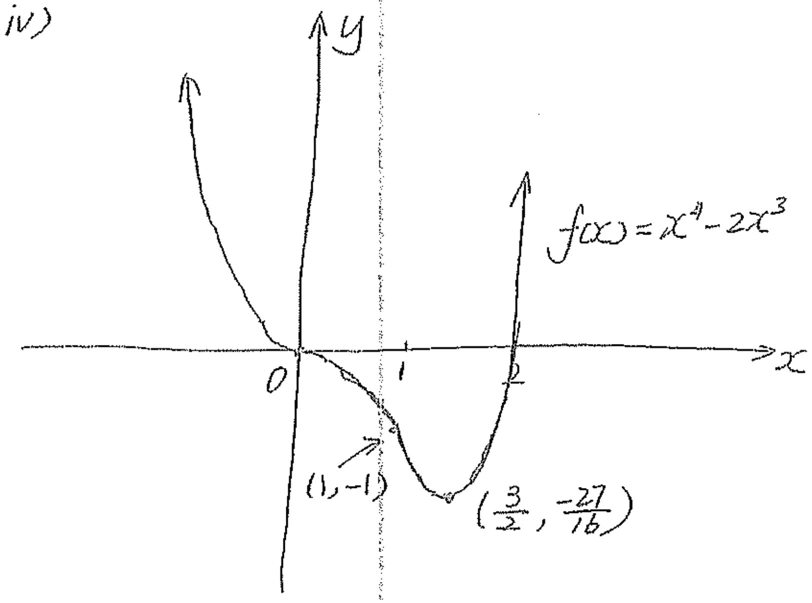
test

x	-0.5	1	0.5
$f''(x)$	−	0	+

1 mark

since there is a change of concavity, $(1,-1)$ is also a point of inflexion

iv)



page 10

Q13 c. i) m of $OA = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad O(0,0)$ $A(-3,1)$

$= \frac{1-0}{-3-0}$

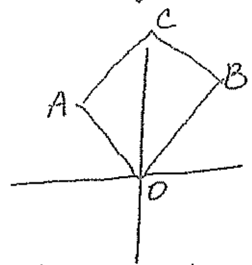
$= \frac{-1}{3}$ 1 mark

ii) m of $OB = \frac{3-0}{1-0} = 3$

$m_{OA} \times m_{OB} = \frac{-1}{3} \times 3 = -1$ 1 mark

since the product of their gradients is −1, therefore OA is perpendicular to OB.

iii)



if BC // OA, then

$m_{BC} = m_{AB} = -\frac{1}{3}$

since BC passes through point B, then

$y - y_B = m_{BC}(x - x_B)$

$y - 3 = -\frac{1}{3}(x-1)$

$y - 3 = -\frac{1}{3}x + \frac{1}{3}$

$y = -\frac{1}{3}x + \frac{10}{3}$

$3y = -x + 10$

rearrange $x + 3y - 10 = 0$

$\therefore$ the equation of BC is $x+3y-10=0$ QED

1 mark correct gradient

1 mark correct proof

iv) sub $x=-2$ into $x+3y-10=0$

$-2+3y-10=0$

$3y = 12 \Rightarrow y = 4 \quad \therefore C$ is $(-2,4)$

Q14

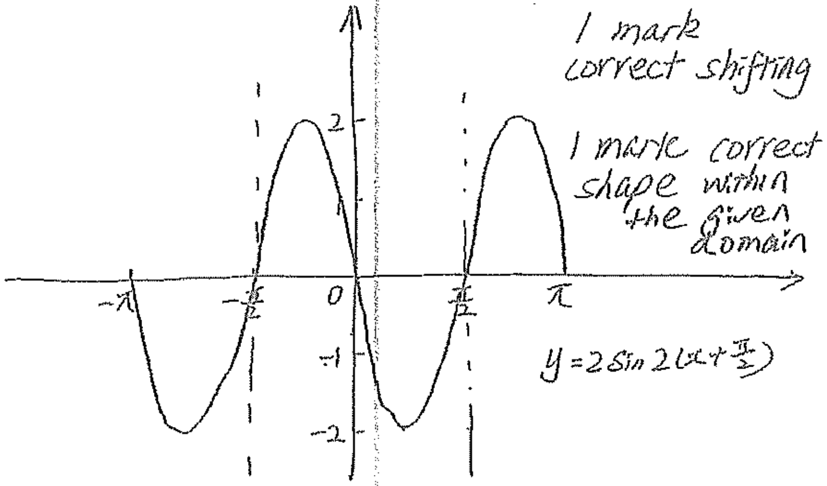
a) $y = 2\sin 2(x + \frac{\pi}{2})$

compare with $y = A\sin(Bx + C)$

amplitude : $A = 2$

period : $P = \frac{2\pi}{2} = \pi$ } 1 mark

phase shift : shift to the left by $\frac{\pi}{2}$



1 mark correct shifting

1 mark correct shape within the given domain

b) $\frac{dx}{dt} = \frac{t}{t^2+1}$ $\quad x(0) = 0$, $x(4) = ?$

$x = \int \frac{t}{t^2+1}\,dt$ — 1 mark correct integration

$= \int \frac{1}{2}\frac{2t}{t^2+1}\,dt$

$x = \frac{1}{2}\ln(t^2+1) + C$ — 1 mark correct evaluation of C

$x(0) = 0 = \frac{1}{2}\ln(1) + C \Rightarrow C = 0$ — 1 mark correct answer

$x(4) = \frac{1}{2}\ln(4^2+1) = 1.42\ m$

Q14

c) i) $y = \sqrt{4 - x^2}$ — 1 mark correct differentiaⁿ

$y = (4 - x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(-2x)(4 - x^2)^{-\frac{1}{2}}$

$y' = \frac{-x}{\sqrt{4 - x^2}}$ — 1 mark ans

ii) $\int \frac{3x}{\sqrt{4-x^2}}\,dx = -3\int \frac{-x}{\sqrt{4-x^2}}\,dx$ — 1 mark

$= -3\sqrt{4-x^2} + C$ — 1 mark

d) To Prove : $\Delta PST \parallel\!| \Delta PQR$

Proof : $\angle SPT = \angle QPR$ (common) — 1 mark

$ST \parallel QR$ (given)

1 mark { $\angle STP = \angle QRP$ (parallel lines, corresponding angles)

$\angle TSP = \angle RQP$ (parallel lines, corresponding angles)

$\therefore \Delta PST \parallel\!| \Delta PQR$ (equiangular)

1 mark final reason

ii) $\because \Delta PST \parallel\!| \Delta PQR$

$\therefore$ All corresponding sides are proportional

$\frac{PR}{PT} = \frac{PQ}{PS}$

$\frac{6+4}{6} = \frac{12}{PS} \Rightarrow PS = 7.2$ — 1 mark

$SQ = PQ - PS = 12 - 7.2 = 4.8$ — 1 mark

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Q15 a) $4^x - 10\times 2^x + 16 = 0$

let $2^x = a$

$(2^x)^2 - 10\times 2^x + 16 = 0$

$a^2 - 10a + 16 = 0$

$(a-2)(a-8) = 0$ — 1 mark

$\therefore a = 2$ or $a = 8$

$2^x = 2$ — $2^x = 8$

$\therefore x = 1$ — $x = 3$

b) i) $\alpha + \beta = \frac{-b}{a}$ — ① $\quad \alpha\beta = \frac{c}{a}$ — ②

let $a = m+2$, $b = m-2$, $c = -2$

$ax^2 + bx + c = 0$

$\alpha = -\beta$ — ③

Sub ③ into ②

$-\beta + \beta = -\frac{m-2}{m+2}$

$0 = \frac{m-2}{m+2} \quad m-2 = 0 \quad m \neq -2$

$\therefore m = 2$

$\therefore m = 2$

ii) $\alpha + \beta = 0 \Rightarrow \alpha = -\beta$ — ①

$\alpha\beta = \frac{-2}{m+2} = \frac{-2}{2+2} = -\frac{1}{2}$ — ②

Sub ① into ② $-\beta^2 = -\frac{1}{2} \Rightarrow \beta = \pm\frac{1}{\sqrt{2}}$

$\alpha = \pm\frac{1}{\sqrt{2}}$

$\therefore \alpha = \frac{1}{\sqrt{2}},\ \beta = -\frac{1}{\sqrt{2}}$

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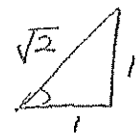
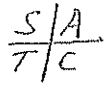
Q15 c. $\sec\theta - 2\cos\theta = 0$

$\frac{1}{\cos\theta} - 2\cos\theta = 0$



$1 - 2\cos^2\theta = 0$

$2\cos^2\theta - 1 = 0$



1 mark $\quad \cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$

1 mark each

$\cos\theta = \frac{1}{\sqrt{2}} \quad \theta = 45°, 315°$ $\quad \therefore \theta = 45°, 135°, 225°, 315°$

$\cos\theta = -\frac{1}{\sqrt{2}} \quad \theta = 135°, 225°$

Q15 d) i) 6% p.a $= \frac{6}{100} \times \frac{1}{12} = 0.005$ per month

$A_1 = (40000 + M) \times 1.005$

$\therefore A_1 = 40000 \times 1.005 + 1.005M$

ii) $A_2 = (A_1 + M) \times 1.005$ — 1 mark

$= (400000(1.005) + 1.005M + M)(1.005)$

$A_2 = 400000(1.005)^2 + 1.005^2 M + 1.005M$

$\therefore A_2 = 400000 \times (1.005)^2 + (1.005 + 1.005^2)M$

iii) $n = 10 \times 12 = 120$

$A_{120} = 400000(1.005)^{120} + M(1.005 + \dots + 1.005^{120})$

but $1.005 + \dots + 1.005^{120} = \frac{1.005(1.005^{120} - 1)}{0.005}$

$r = 1.005$

$= 164.6987$ — 1 mark

$900000 = 400000(1.005)^{120} + M \times 164.6987$

$M = \frac{900000 - 400000(1.005)^{120}}{164.6987}$ — 1 mark

$M = \$1045.80$ — 1 mark

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Q16 a) i) $N = 1000e^{0.005t}$

at $t = 20$

$N = 1000e^{0.005 \times 20}$

$N = 1105.17$

$N = 1105.2$ — 1 mark

ii) $\frac{dN}{dt} = 1000 \times 0.005\, e^{0.005t}$

$= 5e^{0.005t}$ — 1 mark correct differentiation

$= 5e^{0.005 \times 20}$

$\frac{dN}{dt} = 5.526 = 5.5$

b) i) base cost: $27 \times (x \times x) = 27x^2$

other sides cost: $13.5 \times (x \times h \times 4)$

$= 54xh$

$\therefore$ total cost = base cost + other sides cost

$C = 27x^2 + 54xh$

ii) $C = 65.61 = 27x^2 + 54xh$ — \$65.61 = 6561c

$54xh = 6561 - 27x^2$

$h = \frac{6561 - 27x^2}{54x}$

iii) V = base area × height

$= (x \times x) \times h$

$= x^2 h$

$= x^2 \times \frac{6561 - 27x^2}{54x}$ — 1 mark correct substitution

$= \frac{6561x - 27x^3}{54} = \frac{27(243x - x^3)}{54}$

$\therefore V = \frac{1}{2}(243x - x^3)$

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Q16 iv) maximum V occurs when $V' = 0$ and $V'' < 0$

$V = \frac{1}{2}(243x - x^3)$

$V = 121.5x - 0.5x^3$

$V' = 121.5 - 1.5x^2 = 0$

$x^2 = 81$

$x = \pm 9$ — 1 mark for finding $x = 9$

but x is a length, so $x \neq -9$

test $V'' = -3x$ — 1 mark for testing $V'' < 0$

at $x = 9$, $V'' = -3 \times 9 = -27 < 0$

$\therefore$ maximum volume occurs when $x = 9$

$V = \frac{1}{2}(243 \times 9 - 9^3)$ — 1 mark for correct V.

$\therefore V = 729\ cm^3$ is the maximum volume

c. i) $x = 2t - \ln t$

$\dot{x} = 0 = 2 - \frac{1}{t}$

$\frac{1}{t} = 2$

$t = \frac{1}{2}\ s$ — 1 mark for correct time

at $t = \frac{1}{2}s$, $x = 2 \times \frac{1}{2} - \ln\frac{1}{2}$ — 1 mark for position

$x = (1 + \ln 2)\ m \simeq 1.7\ m$

at $t = \frac{1}{2}s$, $\dot{x} = 2 - t^{-1}$ — 1 mark for acceleration

$\ddot{x} = t^{-2} = \frac{1}{t^2} = \frac{1}{(\frac{1}{2})^2} = 4\ m/s^2$

ii) $\lim_{t \to \infty} 2 - \frac{1}{t} = 2 - 0 = 2\ m/s$ — 1 mark

$\lim_{t \to \infty} \frac{1}{t^2} = 0\ m/s^2$ — 1 mark